

# Topos theory

Lecturer: Olivia Caramello

Course for the “Laurea Magistrale in Matematica”  
(first semester of a.y. 2018-2019)

DISAT, Università degli Studi dell’Insubria - Como

## Schedule:

October 2, 2018 - December 18, 2018

Each Tuesday, 16:00-18:00 (Aula Magna, Via Valleggio)

Each Wednesday, 14:00-18:00 (Aula 4.15, Via Valleggio)

# The “unifying notion” of topos

*“It is the **topos** theme which is this “bed” or “deep river” where come to be married geometry and algebra, topology and arithmetic, mathematical logic and category theory, the world of the “continuous” and that of “discontinuous” or discrete structures. It is what I have conceived of most broad to perceive with finesse, by the same language rich of geometric resonances, an “essence” which is common to situations most distant from each other coming from one region or another of the vast universe of mathematical things”.*

A. Grothendieck

Topos theory can be regarded as a **unifying subject** in Mathematics, with great relevance as a framework for systematically investigating the relationships between different mathematical theories and studying them by means of a **multiplicity of different points of view**. Its methods are **transversal** to the various fields and **complementary** to their own specialized techniques. In spite of their generality, the topos-theoretic techniques are liable to generate insights which would be hardly attainable otherwise and to establish **deep connections** that allow effective transfers of knowledge between different contexts.

# The multifaceted nature of toposes

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nature of toposes

A bit of history

Prerequisites and  
exam

Toposes in Como

The role of toposes as unifying spaces is intimately tied to their multifaceted nature; for instance, a topos can be seen as:

- a **generalized space**
- a **mathematical universe**
- a **theory** (modulo a certain notion of equivalence).

The course will start by presenting the relevant categorical and logical background and extensively treat these different perspectives on the notion of topos, with the final aim of providing the student with tools and methods to study mathematical theories from a topos-theoretic perspective, extract new information about correspondences, dualities or equivalences, and establish new and fruitful connections between distinct fields.

# A bit of history

- Toposes were originally introduced by Alexander Grothendieck in the early 1960s, in order to provide a mathematical underpinning for the ‘exotic’ cohomology theories needed in algebraic geometry. Every topological space gives rise to a topos and every topos in Grothendieck’s sense can be considered as a ‘generalized space’.
- At the end of the same decade, William Lawvere and Myles Tierney realized that the concept of Grothendieck topos also yielded an abstract notion of mathematical universe within which one could carry out most familiar set-theoretic constructions, but which also, thanks to the inherent ‘flexibility’ of the notion of topos, could be profitably exploited to construct ‘new mathematical worlds’ having particular properties.
- A few years later, the theory of classifying toposes added a further fundamental viewpoint to the above-mentioned ones: a topos can be seen not only as a generalized space or as a mathematical universe, but also as a suitable kind of first-order theory (considered up to a general notion of equivalence of theories).

# Prerequisites and exam

## Prerequisites:

A Bachelor’s degree in Mathematics (or equivalent mathematical maturity). Although some familiarity with the language of category theory and the basics of first-order logic would be desirable, no previous knowledge of these subjects is required. Indeed, the course will present all the relevant preliminaries as they are needed.

## Exam:

The student will be able to choose between two alternative exam modes:

- Solutions (prepared at home) to exercises assigned by the lecturer and oral presentation on a suitable topic (chosen in agreement with the lecturer) not treated in the course.
- Taking two partial written exams, one administered half way through the course and the other immediately after the end of the course.

# Toposes in Como

The following international event (school + conference) on topos theory has taken place in Como from 24 to 29 June 2018. The school has notably featured lectures by Fields medalists Alain Connes and Laurent Lafforgue.

**SCHOOL: 24-26 JUNE 2018**  
 Lake Como School of Advanced Studies

**School lecturers**  
 Francis Borceux (Université Catholique de Louvain)  
 Olivia Caramello (Università degli Studi dell'Insubria)  
 Alain Connes (IHES)  
 Laurent Lafforgue (IHES)

**CONFERENCE: 27-29 JUNE 2018**  
 Università degli Studi dell'Insubria

**Invited speakers**  
 Tibor Beke (University of Massachusetts)  
 Ingo Blechschmidt (University of Augsburg)  
 Eduardo Dubuc (University of Buenos Aires)  
 Pieter Hofstra (University of Ottawa)  
 Maria Emilia Maietti (Università degli Studi di Padova)  
 Andrew Pitts (University of Cambridge)  
 Mauro Porta (Université de Strasbourg)  
 Bas Spitters (Aarhus University)  
 David Spivak (MIT)

Information and registration:  
[www.tcsc.lakecomoschool.org](http://www.tcsc.lakecomoschool.org)

**TOPOSES IN COMO**

Logos at the bottom include: Università degli Studi dell'Insubria, IEEE, Cariplo, and others.

N.B. The videos of all the talks and related materials will be available soon (check the event website <http://tcsc.lakecomoschool.org/>).