Topos Theory (L24)

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The class of categories known as toposes were first introduced by Alexander Grothendieck in the early 1960s, in order to provide a mathematical underpinning for the 'exotic' cohomology theories needed in algebraic geometry. Every topological space X gives rise to a topos (the category of sheaves of sets on X) and every topos in Grothendieck's sense can be considered as a 'generalized space'.

At the end of the same decade, William Lawvere and Myles Tierney realized that the concept of Grothendieck topos also yielded an abstract notion of mathematical universe within which one could carry out most familiar set-theoretic constructions, but which also, thanks to the inherent 'flexibility' of the notion of topos, could be profitably exploited to construct 'new mathematical worlds' having particular properties.

A few years later, the theory of classifying toposes added a further fundamental viewpoint to the above-mentioned ones: a topos can be seen not only as a generalized space or as a mathematical universe, but also as a suitable kind of first-order theory (considered up to a general notion of equivalence of theories).

Recently, a new view of Grothendieck toposes as unifying spaces in Mathematics being able to serve as 'bridges' for transferring information between distinct mathematical theories has emerged. This approach, first introduced in the lecturer's Ph.D. thesis, has already generated ramifications into distinct areas of Mathematics and points towards a realization of Topos Theory as a unifying theory of Mathematics.

The course will begin by presenting the basic theory of toposes and geometric morphisms, with the aim of reaching these last developments. By the end of the term, the student will have acquired tools and methods to study mathematical theories from a topos-theoretic perspective, extract new information about mathematical dualities, and establish new and fruitful connections between distinct fields.

Pre-requisite Mathematics

Knowledge of the material of the Michaelmas Term course on Category Theory is essential. Some familiarity with classical first-order logic (such as is provided by the Part II course on Logic and Set Theory) would be very desirable but not essential. No previous knowledge of sheaf theory is required.

Literature

1. McLarty, C.: *Elementary Categories, Elementary Toposes*, Oxford U.P. 2002. Recommended for preliminary reading; it's a very gently-paced introduction to the subject, written for philosophers with little mathematical background.
2. MacLane, S. and Moerdijk, I.: *Sheaves in Geometry and Logic: a First Introduction to Topos Theory*, Springer-Verlag, 1992. The best available textbook on the subject, though its approach diverges in some respects from that which will be adopted in the course.
3. Johnstone, P.T.: *Sketches of an Elephant: a Topos Theory Compendium*, Oxford U.P. (2 volumes), 2002. Emphatically not a textbook, but the best place to find out anything you want to know about toposes.
4. Caramello, O.: *The Duality between Grothendieck Toposes and Geometric Theories*, Ph.D. thesis, University of Cambridge, 2009. The place to learn about the view of classifying toposes as unifying spaces and its technical implications.
5. Caramello O.: *The Unification of Mathematics via Topos Theory*, arXiv:math.CT/1006.3930v1, 2010. The programmatic paper on the view 'toposes as bridges'.