

In the paper *Topological Galois Theory* (Advances in Mathematics 291, 646-695 (2016)) Proposition 2.4 is wrongly stated and should be corrected as follows.

**Proposition.** *Let  $G$  be a topological group with an algebraic base  $\mathcal{B}$ . Then*

- (i) *The endomorphisms of the point  $p_G$  can be identified with the element of the projective limit  $\varprojlim_{U \in \mathcal{B}} (G/U)$  of the  $G/U$  for  $U \in \mathcal{B}$ ; in particular, this projective limit has the structure of a monoid.*
- (ii) *The automorphism group of the point  $p_G$  is isomorphic to the group of invertible elements of the monoid  $\varprojlim_{U \in \mathcal{B}} (G/U)$ .*
- (iii) *The group  $G$  is complete if and only if the canonical map from  $G$  to the set of invertible elements of  $\varprojlim_{U \in \mathcal{B}} (G/U)$  is an isomorphism.*
- (iv) *More concretely,  $G$  is complete if and only if for any assignment  $U \rightarrow a_U$  of an element  $a_U \in G/U$  to any subset  $U \in \mathcal{B}$  such that for any  $U, V \in \mathcal{B}$  with  $U \subseteq V$ ,  $a_U \equiv a_V$  modulo  $V$  and there exist elements  $b_U \in G/U$  for  $U \in \mathcal{B}$  such that  $b_U \equiv b_V$  modulo  $V$  whenever  $U, V \in \mathcal{B}$  with  $U \subseteq V$  and  $b_{a_U U a_U^{-1} a_U} \equiv e$ ,  $a_{b_U U b_U^{-1} b_U} \equiv e$  modulo  $U$  for each  $U$ , there exists a unique  $g \in G$  such that  $a_U = gU$  for all  $U \in \mathcal{B}$ .*

**Proof** Since the full subcategory  $\mathbf{Cont}_{\mathcal{B}}(G)$  of the topos  $\mathbf{Cont}(G)$  on the objects of the form  $G/U$  for  $U \in \mathcal{B}$  is dense in  $\mathbf{Cont}(G)$ , the endomorphisms of  $p_G$  correspond exactly to the endomorphisms of the flat functor  $F : \mathbf{Cont}_{\mathcal{B}}(G) \rightarrow \mathbf{Set}$  corresponding to  $p_G$ , that is of the forgetful functor. An endomorphism  $\alpha : F \rightarrow F$  is uniquely determined by the elements  $a_U := \alpha(G/U)(eG) \in G/U$  since the naturality condition for  $\alpha$  with respect to the  $G$ -equivariant arrows  $G/gUg^{-1} \rightarrow G/U$ ,  $g' \rightarrow g'g$ , sending  $e(gUg^{-1})$  to  $gU$  forces  $\alpha(G/U)(gU)$  to be equal to  $a_{gUg^{-1}gU}$  for any  $g \in G$ :

$$\begin{array}{ccc} G/(gUg^{-1}) & \xrightarrow{\alpha(G/gUg^{-1})} & G/(gUg^{-1}) \\ \downarrow & & \downarrow \\ G/U & \xrightarrow{\alpha(G/U)} & G/U \end{array}$$

On the other hand, since any arrow in  $\mathbf{Cont}_{\mathcal{B}}(G)$  can be factored as the composition of a canonical projection arrow of the form  $G/U \rightarrow G/V$  for  $U \subseteq V$  with a canonical isomorphism of the form  $G/gWg^{-1} \rightarrow G/W$ , any assignment  $U \rightarrow a_U$  of an element  $a_U \in G/U$  to any subset  $U \in \mathcal{B}$  such that for any  $U, V \in \mathcal{B}$  with  $U \subseteq V$ ,  $a_U \equiv a_V$  modulo  $V$  defines an endomorphism  $\alpha$  of  $F$  by means of the formula  $\alpha(G/U)(gU) = a_{gUg^{-1}gU}$ . This proves the proposition.  $\square$

Many thanks to Emmanuel Lepage for pointing out the mistake (which has no effects on the rest of the paper) and apologies for any inconvenience caused.